JOURNAL OF SPACECRAFT AND ROCKETS
Vol. 33, No. 6, November–December 1996

Near-Optimal Low-Thrust Orbit Transfers Generated by a Genetic Algorithm

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The effectiveness of a genetic algorithm to design near-optimal low-thrust trajectories is investigated. The orbit transfer is broken into segments and a thrust direction is encoded over each of the segments. A thrust/no-thrust variable allows the trajectory to exhibit coast arcs. Constant and variable thrust propulsion systems are modeled. The thrust profile for a minimum-time Earth-to-Mars constant thrust, and fixed-time Earth-to-Mercury variable thrust trajectories are supplied.

Nomenclature

arc	= total angular displacement of genetic algorithm trajectory, rad
arcf	= desired total angular displacement of the trajectory, rad
(ga)	= final value of a parameter determined by the genetic algorithm
I_{sp}	= specific impulse, s
m_0	= initial mass of the spacecraft, kg
$\dot{m}$	= fuel consumption rate, kg/s
P	= available power, W
P_0	= available power at one astronomical unit (AU; 1.496×10^8 km), W
P/P_0	= relative available power, W
p_c	= probability of crossover, dimensionless
p_m	= probability of mutation, dimensionless
R	= distance from the sun, AU
r	= radial distance of the spacecraft from the attracting center, AU
T	= thrust magnitude, N
t	= time, s
(t_f)	= target final value of a parameter
tt	= total trip time of genetic algorithm trajectory, days
ttf	= acceptable total trip time of genetic algorithm trajectory, days
u	= radial component of spacecraft velocity, AU/time unit (TU; 58.13 days)
v	= tangential component of spacecraft velocity, AU/TU
θ	= angular displacement about the attracting center, deg
μ	= gravitational constant of the attracting center, 1 AU^3/TU^2
ϕ	= thrust direction angle, deg

Introduction

MANY current and future space missions plan to use low-thrust propulsion systems. These propulsion systems offer several advantages over the more conventional high-thrust systems, including increased performance that may lead to smaller spacecraft and possibly smaller and cheaper launch vehicles. Low-thrust missions do, however, provide greater challenges in trajectory design. The traditional chemical rocket has a high-thrust level and is only fired for brief intervals throughout a mission (typically less than 1% of the total mission time). These brief bursts of energy are modeled as impulses. This operational scenario can be used to simplify the equations of motion and allows for a number of techniques to be applied to mission design. In contrast, low-thrust engines, such as a solar electric engine, typically thrust over half of the mission life. Determining the thrust magnitude and direction at each moment of time along the transfer can be difficult.

The genetic algorithm's (GA's) usefulness for solving impulsive trajectories is well documented.[1–3] The purpose of this study was to investigate the GA's effectiveness at determining low-thrust trajectories. Two trajectories are presented: an Earth-to-Mars constant thrust and an Earth-to-Mercury variable thrust mission. The Earth-to-Mars mission is a minimum-time transfer between two circular orbits and the Earth-to-Mercury transfer is time fixed and uses real ephemeris data.

Problem Definition

There are many classes of orbit transfers. In general, they can be from planet to planet, from one orbit to another around a planet, or even from a planet to another celestial body such as a comet. This study considers low-thrust trajectories from one planet to another. An initial test problem with a known solution was used to aid in the selection of the GA parameters and the verification of the solutions found by the GA. Bryson and Ho[4] give an analysis of a minimum-time transfer from Earth to Mars and provide a numerical solution for a particular spacecraft configuration. The initial test problem assumes circular orbits for Earth and Mars. In addition, the parameters of the problem have been transformed to canonical units.[5] The geometry of the orbit transfer is illustrated in Fig. 1. The attracting center is the sun. The dashed circles represent the orbits of Earth and Mars. Because the spacecraft begins in the Earth's orbit, $r(0)$

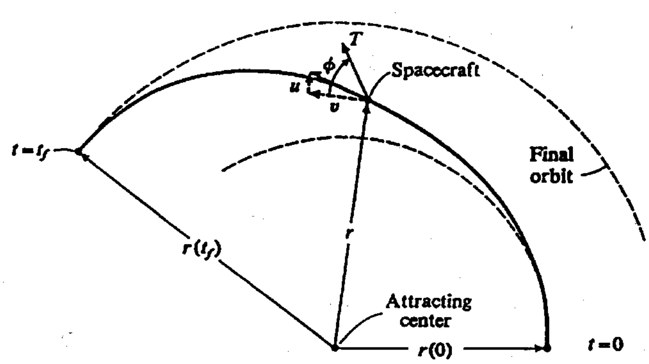

Fig. 1 Geometry of an Earth-to-Mars orbit transfer (reprinted with permission from Ref. 4).

Received Jan. 10, 1996; revision received June 24, 1996; accepted for publication June 28, 1996.

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is equal to 1 astronomical unit (AU) for this simplified case. The actual parameters for the orbits can be found in a standard orbital mechanics book.[5]

The equations of motion that describe a spacecraft with a low-thrust propulsion system are coupled nonlinear differential equations that have problem specific boundary conditions. These equations include the thrust angle ϕ and thrust magnitude T, which are referred to as the control input. Given the control input of the trajectory, the equations can be integrated. The equations of motion for a low-thrust orbit transfer in a plane are given in Ref. 4 as

$$\dot{r} = u \tag{1}$$

$$\dot{\theta} = v/r \tag{2}$$

$$\dot{u} = \frac{v^2}{r} - \frac{\mu}{r^2} + \frac{T\sin(\phi)}{m_0 - \dot{m}t} \tag{3}$$

$$\dot{v} = -\frac{uv}{r} + \frac{T\cos(\phi)}{m_0 - \dot{m}t} \tag{4}$$

The initial conditions, $r(0)$, $\theta(0)$, $u(0)$, and $v(0)$, correspond to the spacecraft's position and velocity while in the Earth's orbit at the initiation of the transfer. Note that for orbit transfer problems, Eq. (2) can be removed from the problem formulation since the final angular displacement is not specified. The initial conditions for the problem just discussed are

$$r(0) = 1.0, \qquad \theta(0) = 0.0, \qquad u(0) = 0.0, \qquad v(0) = 1.0 \tag{5}$$

where $r(0)$ is in AU and $u(0)$ and $v(0)$ are in AU/time unit (TU). Likewise, the terminal boundary conditions corresponding to a circular Mars orbit are

$$r(t_f) = 1.524, \qquad \theta(t_f) = \text{free}, \qquad u(t_f) = 0.0 \qquad v(t_f) = 0.81 \tag{6}$$

The final angular displacement, $\theta(t_f)$, is free to be chosen since an orbit-to-orbit transfer is being modeled in the test problem; however, in the Earth-to-Mercury rendezvous presented later, a $\theta(t_f)$ will be specified. Also, the thrust and mass-related terms were chosen to be consistent with the problem in Bryson and Ho[4]:

$$T = 3.787 \tag{7}$$

$$m_0 = 4545.5 \tag{8}$$

$$\dot{m} = 6.787e - 5 \tag{9}$$

For this test problem, the propulsion system modeled contains a fixed power source such as a nuclear electric engine. The minimum time required to perform this orbit transfer is 193 days.[4]

Genetic Algorithm

Genetic algorithms are robust parameter optimization techniques based on the Darwinian concept of natural selection. A GA differs from calculus-based optimization algorithms that are commonly used in low-thrust spacecraft mission design in that the GA iterates on a set of solutions toward the optimum instead of a single solution. Each solution is termed an individual, and each set of solutions is referred to as a generation or population. Iteration is performed from one generation to the next. Every individual in a generation is assigned a fitness that corresponds to its performance based on an objective function for that particular problem. Typically, the fitness is a function of quantities that are to be maximized or minimized on the optimal solution. An individual that performs well for a given generation is assigned a high fitness. Characteristics from the most fit individuals are then used to help define the next generation of individuals resulting in the computational analog of the survival of the fittest. The GA provides an efficient parameter search algorithm that exploits information from previous generations to create new individuals with expected improved performance.

Table 1 GA parameters for Earth-to-Mars transfer

Parameter	Value
Population size	150
Generations	50
Crossover probability	0.8
Mutation probability	0.005

The basic GA used for this study included tournament selection, single-point crossover, jump mutation, and elitism. Tournament selection is a reproduction method that randomly selects two individuals from the current generation and compares their associated fitness. The individual with the better fitness is allowed to reproduce. The tournament selection continues until the appropriate number of parents have been identified. Next, a crossover of information occurs. Single-point crossover is used and requires that two parents be randomly chosen to be paired. If crossover of information is to happen at a given location, based on the probability of crossover, that bit and the subsequent bits in each parent are swapped. Mutation is the third probabilistic operator. Mutation involves the switching of a randomly selected bit from a 1 to a 0 or vice versa. The occurrence of mutation, based on the probability of mutation, models natures ability to have species evolve beneficial characteristics over several generations. The operator known as elitism copies the best individual from the previous generation into the new generation if a better individual was not created in the new generation, i.e., elitism was chosen to prevent the current best solution from being lost.

The optimization goals for this GA are to choose a thrust history, which includes the thrust direction and magnitude, such that the boundary conditions are satisfied to an acceptable level and to provide these solutions in a reasonable amount of time. For the fixed-time transfers, only two control variables per trajectory segment need to be coded into the GA; the thrust angle and a variable indicating whether the engines are on or off, since a coast may be part of the trajectory. This second variable is called tswitch. The trajectory was divided into 10 or 20 segments, which resulted in a variable string that consisted of 19 parameters (10 angles, 9 tswitches), or 39 parameters (20 angles, 19 tswitches). Since the engines are required to be on during the first segment to initiate the transfer, only the thrust angle is needed during that segment. This string of parameters is coded into a binary string consisting of 69 bits for the 10 segment transfers and 139 for the 20 segment cases. The thrust angle is mapped into 6 bits and is equally distributed over 360 deg. The tswitch is coded as a single bit with 1 representing a thrust segment and a 0 representing a coast. These parameters were located within the string by segment such that the variables alternated between thrust angle and tswitch.

Four problem-dependent GA parameters (the number of individuals, the number of generations, the crossover probability rate, and the mutation probability rate) were varied to determine their effect on the initial test case.[6] Table 1 displays the values that are used in the following Earth-to-Mars transfer. These parameters are consistent with the recommended standard settings.[7] The GA was programmed in Fortran and run on a Convex C240. The Fortran code was principally a translation of the Pascal code found in Goldberg's text.[7] The equations of motion were integrated using an International Mathematical and Statistical Library integrator. The equations for the first segment were integrated using the initial conditions, and the resulting parameter values became the initial conditions for the integration of the next segment. This process continues until all segments are evaluated.

Earth-to-Mars Transfers

A 10 segment Earth-to-Mars constant thrust minimum-time orbit transfer is presented. The time optimization added one 5-bit binary parameter to represent the total transfer time, and this time must be bound from above and below. The GA fitness function is a combination of the transfer time and the final boundary conditions. The boundary conditions were all scaled to be within 1%. The transfer time bounds were determined iteratively by the following technique. A flight time was chosen to be large, say, 250 days, and a fixed-time transfer was solved. If all of the boundary conditions were satisfied within their acceptable limits and there was significant amount of coasting, then the upper bound of the flight time was reduced. Once the transfer time resulted in thrusting for most of the segments, this time was defined as the acceptable transfer time, which for this trajectory was found to be 197 days. The transfer time was then allowed to vary about the acceptable transfer time: for this example, from 190 to 221 days in increments of 1 day. The fitness function is

$$\text{fitness} = -\left\{\frac{[r(ga) - r(t_f)]^2}{(0.01)^2} + \frac{[u(ga) - u(t_f)]^2}{(0.01)^2} + \frac{[v(ga) - v(t_f)]^2}{(0.01)^2} + \frac{(tt - ttf)^2}{(3.5)^2}\right\} \tag{10}$$

The optimal solution that the GA solution will be compared with has the boundary conditions shown in Table 2 and has a total transfer time of 193 days.[4] Figure 2 shows the known optimal trajectory[4] with the thrusting angle given every 19.3 days.

The fitness function of Eq. (10) along with the parameter values of Table 2 were very successful at finding near-optimal results. A typical result from one of the runs is shown in Fig. 3. The inner arc is the Earth's orbit and the outer arc represents Mars' orbit. The trajectory is shown with the respective thrust angles. Coast arcs are represented by a hollow circle at the beginning of the segment. The thrust angles shown are held constant throughout the segment. This trajectory had a total trip time of 199 days, 6 days greater than the optimal solution, but surprisingly consisted of a coast of nearly 40 days during the fourth and fifth segments. The final conditions of the solution shown in Fig. 3 are given in Table 3. The boundary conditions were met to within 1%. The 1% convergence criterion was arbitrarily chosen. If this accuracy is not sufficient, the solution can be refined by running additional generations with additional segments. Comparing Figs. 2 and 3, the most questionable region of the GA thrust histories occurs in the middle of the transfer. In this region, the GA could not apply enough selection pressure to smooth the thrust histories. A partial explanation of this lies simply in the location of the segments within the middle of the transfer. The beginning and the end of the transfer are critical to the development of the solution. As the solution begins to grow in the early generations, potential solutions whose first few segments are completely wrong have no chance of satisfying the final boundary conditions regardless of the remaining part of the trajectory. As a result, they are quickly eliminated. Likewise, potential solutions with poor final segments are eliminated because certain boundary conditions, such as radial velocity, are very sensitive to these segments. This leaves a population of mostly good trajectory beginnings and endings and a variety of segments in the middle.

Table 2 Initial and final boundary conditions for Earth-to-Mars transfer

	$r(t)$, AU	$\theta(t)$, deg	$u(t)$, AU/TU	$v(t)$, AU/TU
Initial conditions	1.000	0.0	0.000	1.000
Final conditions	1.524	open	0.000	0.810

Table 3 Final boundary conditions achieved by the best GA solution

	$r(t)$, AU	$\theta(t)$, deg	$u(t)$, AU/TU	$v(t)$, AU/TU
Final conditions	1.512	149.5	0.008	0.802

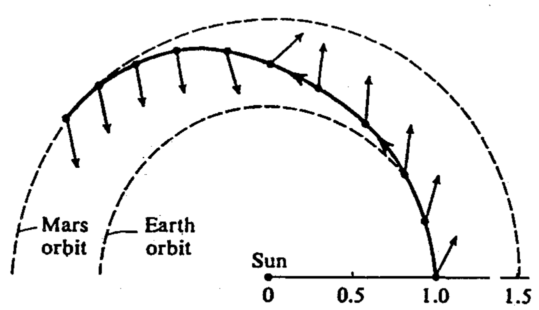

Fig. 2 The optimum low-thrust transfer (reprinted with permission from Ref. 4).

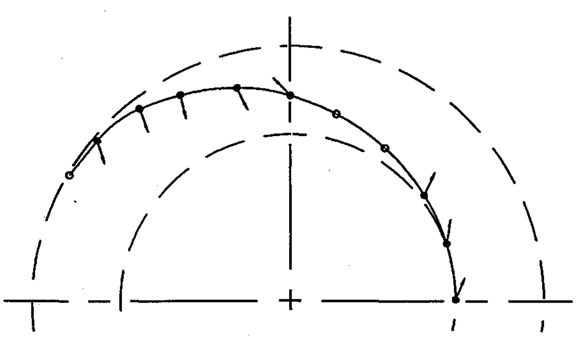

Fig. 3 Near-optimal Earth-to-Mars transfer found by the GA.

Earth-to-Mercury Transfers

A 20 segment Earth-to-Mercury time-fixed solar electric propulsion (SEP) transfer is discussed. The SEP is based on a solar array model from the Jet Propulsion Laboratory.[8] The relative solar array power is given by

$$\frac{P}{P_0} = \frac{1}{R^2}\left[\frac{1.4279 - (0.6139/R) + (0.0038/R^2)}{1 - 0.2619R + 0.0797R^2}\right] \tag{11}$$

This equation was developed by fitting data of solar arrays designed for a mission to an inner planet. This particular array has a maximum relative power of 1.35 at a heliocentric distance of 0.65 AU. The arrays would be angled away from the sun when the spacecraft is within 0.65 AU so as to maintain the power at a constant relative level of 1.35 (Ref. 8).

The arrays were sized so that the power produced at 1 AU with thrusters that had a specific impulse of 3000 s produced a thrust level equivalent to the value given in Eq. (7). The thrust is proportional to the available power,

$$T = 2P/gI_{sp} \tag{12}$$

The fuel consumption rate

$$\dot{m} = T/gI_{sp} \tag{13}$$

was variable but directly proportional to the thrust. The initial mass was 6818.3 kg. The fitness function was changed to a positively increasing function, which for this problem produced better results than simply adding the penalty functions, as was used in the previous trajectory:

$$\text{fitness} = 1\bigg/\left\{\frac{[r(ga) - r(t_f)]^2}{(0.01)^2} + \frac{[u(ga) - u(t_f)]^2}{(0.01)^2} + \frac{[v(ga) - v(t_f)]^2}{(0.01)^2} + \frac{(\text{arc} - \text{arcf})^2}{(2.0)^2}\right\} \tag{14}$$

The last term in the fitness function represents the angular displacement portion of the fitness, where arc is the total angular displacement of the GA derived trajectory and arcf is the desired total angular displacement. The transfer begins on May 6, 1997, and lasts for 355 days. The resulting initial and final boundary conditions are presented in Table 4 and are the actual planar coordinates for the Earth and Mercury on the departure and arrival dates. Note that the angular displacement of Mercury is over two complete revolutions.

The GA parameters selected to solve this problem are given in Table 5. There was only one change from the baseline parameters;

Table 4 Initial and final boundary conditions (ephemeris data)

	$r(t)$, AU	$\theta(t)$, deg	$u(t)$, AU/TU	$v(t)$, AU/TU
Initial conditions	1.009	0.0	0.014	0.991
Final conditions	0.467	749.9	−0.0001	1.297

Table 5 GA parameters for Earth–Mercury SEP transfer

Parameter	Value
Population size	150
Generations	75
Crossover probability	0.8
Mutation probability	0.005

Table 6 Final boundary conditions achieved by the best GA solution

	$r(t)$, AU	$\theta(t)$, deg	$u(t)$, AU/TU	$v(t)$, AU/TU
Final conditions	0.471	750.2	0.001	1.296

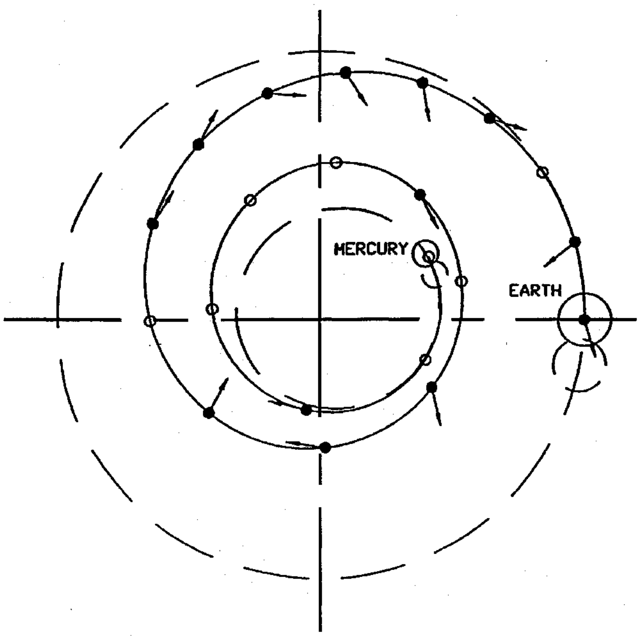

Fig. 4 Fixed-time Earth-to-Mercury transfer found by the GA.

the number of generations was increased to 75 because of the increase in the number of segments to 20.

The resulting trajectory determined by the GA is shown in Fig. 4. The large inner and outer curves represent the orbits of Mercury and Earth around the sun, respectively. The planets are shown in their orbits by smaller solid and dashed marks. The eccentricity of Mercury's orbit is easily noticed. The solid circle for Earth represents its initial location, and the dashed circle represents its location at the end of the transfer. The dashed circle for Mercury represents its initial location, and the solid circle represents its location at the end of the transfer. The final boundary values achieved by the transfer are presented in Table 6. This solution demonstrates a pattern of backward thrusts throughout the trajectory, and there are five coast arcs totaling nearly 125 days. This clearly shows that the SEP spacecraft had more than sufficient power for this transfer. Even though the total flight time was fixed at 355 days, the spacecraft coasted alongside Mercury for over 20 days at the end of the transfer.

Three other near-optimal Earth-to-Mercury trajectory cases were tested with the GA. These included a 20 segment Earth-to-Mercury constant thrust time-fixed transfer with real ephemeris data, a 20 segment Earth-to-Mercury minimum-time constant thrust transfer, and a 20 segment Earth-to-Mercury minimum-time SEP transfer. For the two minimum-time cases, both Earth and Mercury were assumed to be in circular orbits about the sun. The GA had no difficulty finding near-optimal solutions for any of these cases (not shown; see Ref. 6) and in general the trajectories looked similar to that illustrated in Fig. 4.

The Earth-to-Mercury transfer tested the GA's accuracy over large segments and diverse ephemeris data. The time-fixed transfer exhibited the expected exponential spiral appearance traditionally seen in low-thrust trajectories.[9] The variable thrust transfer exhibited greater randomness at the beginning of the trajectory than was expected. However, the propulsion system, which was solar electric, gained power to a maximum level at 0.65 AU and maintained that level for the remainder of the trajectory. As a result, the last 10 segments made a significantly greater contribution to the transfer than the first 10 segments, which leads to greater selection pressure applied to these segments and, therefore, greater attention applied by the GA.

The examples shown illustrate the power of the GA to determine the general characteristics of low-thrust trajectories. Refinement of the solutions can result by allowing the GA to evolve over more generations and by increasing the number of segments on the trajectory (decrease the integration time per segment. The GA can provide a standalone mission planning tool or could be used to generate the initial guess that is required by direct optimization techniques such as direct collocation and differential inclusion methods.[9] Also, preliminary tests indicate that the GA can be used to integrate the Euler–Lagrange equations, thereby providing optimal solutions.

Conclusions

The goal of this study was to use a GA to find near-optimal low-thrust trajectories. Minimum-time transfers were designed along with fixed-time transfers that matched ephemeris data. Also, two low-thrust propulsion systems were modeled. Throughout this study the GA provided solutions that could be used for preliminary mission planning.

Acknowledgments

Many thanks to D. Carroll and D. Goldberg for several useful genetic algorithm tips, insights, and conversations.

References

[1]Gage, P. J., Braun, R. D., and Kroo, I. M., "Interplanetary Trajectory Optimization Using a Genetic Algorithm," *Journal of Astronautical Sciences*, Vol. 43, No. 1, 1995, pp. 59–75.

[2]Carter, M. T., and Vadali, S. R., "Parameter Optimization Using Adaptive Bound Genetic Algorithms," American Astronomical Society, AAS Paper 95-140, Feb. 1995.

[3]Pinon, E., III, and Fowler, W. T., "Lunar Launch Trajectory Optimization Using a Genetic Algorithm," American Astronomical Society, AAS Paper 95-142, Feb. 1995.

[4]Bryson, A. E., and Ho, Y. C., *Applied Optimal Control*, 1st ed., Hemisphere, New York, 1975, pp. 66–69.

[5]Bate, R. R., Mueller, D. D., and White, J. E., *Fundamentals of Astrodynamics*, 1st ed., Dover, New York, 1971, pp. 40, 41.

[6]Rauwolf, G. A., "Near-Optimal Low-Thrust Orbit Transfers Generated by a Genetic Algorithm," M.S. Thesis, Aeronautical and Astronautical Engineering Dept., Univ. of Illinois at Urbana–Champaign, IL, Aug. 1995.

[7]Goldberg, D. E., *Genetic Algorithms in Search, Optimization and Machine Learning*, Addison–Wesley, Reading, MA, 1989.

[8]Sauer, C. G., Jr., "Modeling of Thruster and Solar Array Characteristics in the JPL Low-Thrust Trajectory Analysis," AIAA Paper 78-645, April 1978.

[9]Coverstone-Carroll, V. L., and Williams, S., "Optimal Low-Thrust Trajectories Using Differential Inclusion Concepts," *Journal of Astronautical Sciences*, Vol. 42, No. 4, 1994, pp. 379–393.

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